**Integration by Substitution (Sec. 5.5)**

**DEFINITION – Chain Rule**

*(Convention: F(x) is the derivative of f(x))*

Examples

Here, “e^x” is f and cos x is g

Here, is f, and 1 + x^3 is g

**DEFINITION – Integration by Substitution**

Using the integral definition above,

Let u = g(x) (Therefore *u’ = g’(x)*, , )

So we get,

*(Because g’(x) = g’(x) du)*

*(Because u = g(x))*

To do integration by substitution, you need to have at least *a constant multiple* of .

Examples

Let



Antiderivatives of non-trivial trigonometric functions (more examples)

Definite Integrals

**DEFINITION – Substitution Rule for Definite Integrals (p. 411)**

Examples

There are two ways we could attack this:

1. Use substitution to find the antiderivative then take the difference at the two points;
2. Make a complete change of variable

a would work like:

b would work like:

Another example is:

*Let u = x^2*

*(Note that this f’n is range-restricted between –pi/2 and pi/2)*

And now for something completely different…

There is a potential “trick” for integration, using **symmetry**.

**DEFINITION – Even Function**

Examples (constant multiples also work):

Properties

* f(0) = 0
* Limits approaching either infinity are equal to the infinity of the sign of the constant multiple.

**DEFINITION – Odd Function**

Examples (constant multiples work):

Properties

* Limits approaching either infinity are opposite.
* = 0

**DEFINITION – Symmetric**

A **symmetric** function is a function that is *either* even or odd.

Properties

* f(0) = 0

See examples 10e, 11